





















## Example 1

• A 50.0 kg crate is pulled 40.0 m along a horizontal floor by a constant force of 100.0 N at an angle of 37° from the horizontal. Calculate the work done on the crate.

 $W = Fd\cos\theta$ 

$$W = (100)(40)\cos 37$$

W = 3190 J



$$W = Fd \cos \theta$$
$$W = (F_{app} - F_f)d$$
$$F_f = F_{app} - \frac{W}{d}$$
$$F_f = 160 - \frac{50}{5}$$
$$F_f = 150 \text{ N}$$

#### Energy

- When work is done on a system, energy is added to the system.
- The energy could be stored (potential energy) or cause motion (kinetic energy).
- Work done by friction leaves the system in the form of heat.





- The net force arises from the applied force  $F_{app}$  and the friction force  $F_f$ .
- Since the net force is parallel to the displacement, the net work is

$$W_{net} = F_{net}d$$

- The result of the net force is an acceleration from  $v_0$  to v.
  - The kinetic energy will increase.
- Substituting for  $F_{net}$  gives

 $W_{net} = mad$ 

• We can use a kinematic equation to substitute for acceleration.

$$v^{2} = v_{0}^{2} + 2a(x - x_{0}) \qquad x - x_{0} = d$$
$$a = \frac{v^{2} - v_{0}^{2}}{2d}$$
$$W_{net} = m\left(\frac{v^{2} - v_{0}^{2}}{2d}\right)d$$

Rearranging this equation gives

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

- This expression is called the work-energy theorem, and it applies even for forces that vary in direction and magnitude.
- The theorem implies that the net work on a system equals the change in the quantity  $\frac{1}{2}mv^2$ .

• The quantity  $\frac{1}{2}mv^2$  in the work-energy theorem is defined to be the translational kinetic energy.

$$K = \frac{1}{2}mv^2$$

• Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

#### Example

- A 145 g baseball is thrown with a speed of 25 m/s.
  - a) What is the kinetic energy of the ball?
  - b) How much work was done on the ball to make it reach this speed if it started from rest?

a)  $K = \frac{1}{2}mv^2$  $K = \frac{1}{2}(145 \times 10^{-3})(25)^2 = 45 \text{ J}$ 

b) 
$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$
  
 $W_{net} = 45 \text{ J}$ 











- This expression applies for small distances where the value of *g* remain constant.
- We need an expression that works over distances such that *g* is not constant.
- This is necessary to correctly calculate the energy needed to place satellites in orbit or to send them on missions in space.
- We can calculate the work done (and thus potential energy) by finding the area under a force vs distance curve.





We define gravitational potential energy as

$$U_G = -\frac{Gm_1m_2}{r}$$

• Note:

- Force at an infinite distance is 0.
- Therefore, potential energy at an infinite distance must be 0.
- Potential energy decreases as the distance decreases.

### Potential Energy

- Potential energy is the energy a system has due to position, shape, or configuration.
- It is stored energy that is completely recoverable.

#### **Conservative Forces**

- A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.
- We can define a potential energy for any conservative force.
- The work done against a conservative force to reach a final configuration is the potential energy added.

#### **Elastic Potential Energy**

- The work done to stretch a spring a distance *x* is *W* = *Fx*.
- However, the force is not constant.
  - Hooke's law:  $|\vec{F}_s| = k |\vec{x}|$
- The work, therefore, is the area under the force vs displacement curve for the spring.



• The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy.

$$\Delta K = \Sigma W_i = \Sigma F_{\parallel,i} d_i$$

## Conservation of Mechanical Energy

• If a conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy.

$$-\Delta U = \Delta K$$
$$-(U_f - U_i) = K_f - K_i$$
$$K_i + U_i = K_f + U_j$$

- This means that the total kinetic and potential energy is constant for any process involving only conservative forces.
- This is known as the conservation of mechanical energy principle.
  - The total kinetic plus potential energy of a system is defined to be its mechanical energy.





$$K_{i} + U_{i} = K_{f} + U_{f}$$

$$\frac{1}{2}mv^{2} = mg\Delta y + \frac{1}{2}kx^{2}$$

$$x = \sqrt{\frac{\frac{1}{2}mv^{2} - mg\Delta y}{\frac{1}{2}k}}$$

$$x = \sqrt{\frac{\frac{1}{2}(0.5)(3)^{2} - (0.5)(9.8)(0.25)}{\frac{1}{2}(20)}} = 0.32 \text{ m}$$



# Nonconservative Forces

- A nonconservative force is one for which work depends on the path taken.
- Friction is a nonconservative force.
  - Work done against friction depends on the length of the path between the starting and ending points.



- Because of this dependence on path, there is no potential energy associated with nonconservative forces.
- Work done by a nonconservative force adds or removes mechanical energy from a system.
  - Friction creates thermal energy that dissipates, removing energy from the system.
  - Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.
- The energy dissipated by friction is typically equated to the force of friction times the length of the path over which the force is exerted.

 $\Delta E_{mech} = F_f d \cos \theta$ 

• When nonconservative forces are acting on the system the work-energy theorem states that the change in mechanical energy is equal to the work done by the nonconservative forces.

$$W_{nc} = \Delta K + \Delta U$$
$$W_f = (K_f - K_i) + (U_f - U_i)$$
$$K_i + U_i + W_f = K_f + U_f$$
$$K_i + U_i - F_f d \cos \theta = K_f + U_f$$





$$K_i + U_i + W_f = K_f + U_f$$
$$\frac{1}{2}mv^2 - F_f d = mg\Delta y$$
$$\frac{1}{2}mv^2 - F_f d = mgd\sin\theta$$
$$d = \frac{\frac{1}{2}mv^2}{mg\sin\theta + F_f}$$
$$d = \frac{\frac{1}{2}(1)(2.5)^2}{(1)9.8\sin 10 + (5)} = 0.47 \text{ m}$$

# Law of Conservation of Energy

• Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

$$\Sigma E_i = \Sigma E_f$$









$$P_{avg} = \frac{\Delta E}{\Delta t}$$

• We can derive an equation to determine the instantaneous power delivered to an object by a constant force parallel to velocity of the object.

$$P_{inst} = \frac{W}{\Delta t} \qquad \begin{array}{l} W = F_{\parallel}d \\ W = Fd\cos\theta \end{array}$$
$$P_{inst} = \frac{F_{\parallel}d}{\Delta t} = \frac{Fd\cos\theta}{\Delta t}$$
$$P_{inst} = F_{\parallel}v = Fv\cos\theta$$



$$P = \frac{\Delta E}{\Delta t} \qquad \Delta E = \left(K_f + U_f\right) - \left(K_i + U_i\right)$$
$$P = \frac{K_f + U_f}{\Delta t} = \frac{\frac{1}{2}mv^2 + mg\Delta y}{\Delta t}$$
$$P = \frac{\frac{1}{2}(60)(2)^2 + (60)9.8(3)}{(3.5)} = 538 \text{ W}$$

# Efficiency

- Even though energy is conserved in an energy conversion process, the output of useful energy or work will be less than the energy input.
- The efficiency of an energy conversion process is defined as

$$Efficiency = \frac{useful \ energy \ or \ work \ output}{total \ energy \ in} = \frac{W_{out}}{E_{in}}$$